# Arithmetic After School: How do Adults' Mental Arithmetic Abilities Evolve with Age? 

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#### Abstract

To date, few studies have investigated the evolution of problem solving and general numeracy abilities during adulthood: skills that have obvious social importance. In this research, evolutions in adults' mental arithmetic skills were investigated using data from the IVQ 2004 French national survey, which tested 9,185 adults aged between 18 and 65. Whereas some of our results confirm previous work, others are more surprising. For example, participants with higher levels of education achieved higher scores than did participants with lower levels of education, and this was true at all ages; however, improvements with age in performance only were found for participants with low levels of education (first grade or lower). For higher levels of education, performances were stable. Our results suggest that lifespan mental arithmetic, a domain not affected by the Flynn effect, relies mainly on pragmatic cognition, and develops as a result of everyday activities, as well as through initial instruction.


In most societies, adults regularly have to undertake mental calculations connected with tasks such as calculating prices, evaluating reductions, comparing sizes, and the like. As this everyday problem-solving activity falls into the domain of intellectual functioning, it may provide fertile ground for studying cognitive development throughout life (i.e., lifespan). Unfortunately, very little lifespanrelated data on mathematical problem-solving abilities have been collected, as, to date, most research has been limited to pure calculations (Lemaire, Arnaud, \& Lecacheur, 2004; Salthouse \& Coon, 1994; Salthouse \& Kersten, 1993; Schaie, 2005) or, for reasons of convenience, researchers

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only have studied the arithmetic skills of adults at the age of conscription (Sundet, Barlaug, \& Torjussen, 2004).

However, a survey carried out in 2004 by the French National Institute of Statistics and Economic Studies (INSEE) ${ }^{1}$ has provided a body of data that can be analyzed in order to evaluate the effect of age on adults' mental arithmetic skills. The objective of this "Information and Daily Life" (Information et Vie Quotidienne - IVQ 2004) survey was to evaluate the "literacy" and "numeracy" of a sample of more than 10,000 adults, chosen to provide a representative cross-section of the population of France. IVQ 2004 was designed to uncover possible links between the skills evaluated and a range of biographical variables (e.g., age, gender, level of education). Our research team was asked to draw up the numeracy evaluation items for the survey (Charron \& Meljac, 2003); other research teams produced the "literacy" items. Although the concept of literacy has been more or less precisely defined in the literature, there is no
we propose that numeracy be defined as the numerical and mathematical skills used in everyday and professional life (Charron \& Meljac, 2003). Everyday situations often involve problem solving; therefore, most of the items created for the survey were mental arithmetic problems to be solved without using a calculator (Charron \& Meljac, 2003). This work led to the establishment of a corpus of data that, to the best of our knowledge, is totally unique (on this scale).

Our objective was to use this corpus to evaluate how adults' everyday mental arithmetic skills, which play an important role in everyday and professional life, evolve with age. Before presenting the survey in greater detail and advancing our hypotheses, we would like to discuss two preliminary considerations. First, it was necessary to ascertain whether data from cross-sectional surveys were suitable for studying the evolution of arithmetic abilities, even if the difficulty in isolating specific factors (such as age) cannot be denied. This was undertaken by analyzing two particularly relevant studies. Second, we felt that no investigation of mental arithmetic abilities could be conducted without examining the cognitive foundations of arithmetic skills. This was undertaken with reference to the now classic distinction between pragmatic cognition (i.e., crystallized; Baltes, Staudinger, \& Lindenberger, 1999) and mechanical cognition (i.e., fluid; Baltes et al.). Therefore, we believed that for this type of study it was essential to carry out an empirical examination of development.

## Age-related Evolution or Flynn Effect?

Our first concern was to determine whether a cross-sectional survey of the population could be used to study the effect of age on mental arithmetic abilities. In contemporary lifespan studies, the dates of birth of the participants cover almost the whole of the 20th century; therefore, every study population will include a number of different cohorts-for example, war babies, baby-boomers, and school students who experienced the 1970 French "modern math" reform. It is known that the influence of the cohort in cross-sectional studies cannot be statistically differentiated from the effects of age (Schaie, 2005). As a result it is difficult to determine which elements of development are due to age. Furthermore, the results of cognitive tests for cohorts with the same age but born at different periods are subject to a well-known phenomenon called the Flynn effect in which the most recent cohort will always obtain the highest scores (for a review, see Flieller, 2001). Therefore, before attempting to use cross-sectional survey data to study the evolution of arithmetic abilities, we had to determine whether
mental arithmetic was subject to the Flynn effect. If this had been the case, and if the data had shown a decrease in performance with age, we would not have been able to draw any conclusions about the effect of aging, because the decrease may just have been a reflection of the Flynn effect. However, the two studies presented below suggested that mental arithmetic abilities were not susceptible to the Flynn effect.

The first of these studies, by Sundet et al. (2004), tested similarly aged, male Norwegian conscripts over a period of almost 50 years. Because the tests were conducted between the mid-1950s and 2002, the dates of birth of the cohorts were between 1935 and 1984 (approximately). The tests administered by Sundet et al. included a time-limited ( 25 -minute) arithmetic test consisting of 30 items presented in written form, which was designed to measure logical reasoning, as well as arithmetic and algebra skills. According to the authors, this test was comparable with the arithmetic sub-test of the Wechsler Adult Intelligence Scale (WAIS ; Wechsler, 1956), which includes mental arithmetic problems. The results of Sundet et al.'s study, which are summarized in Figure 1, suggested that mental arithmetic abilities were not subject to the Flynn effect, as performances did not increase during the most recent observation years. Nevertheless, Sundet et al.'s (2004) study was very specific, in that it involved an examination of Norwegian conscripts, and its conclusions are debatable, as the math tests were changed during the study.

A second study to throw light on this subject was carried out by Schaie (2005). Based on an exceptional data set compiled over almost one half a century, Schaie's work allows us to review the situation with respect to numerical abilities (at least addition-a skill that is needed to solve most arithmetic problems without using a calculator). Our examination of performance as a function of birth cohort was based on Schaie's cross-sectional data, in which participants were given a limited time (6 minutes) to check whether the solutions given for 60 simple additions were correct. The data obtained by Schaie for the two extreme ages (25 and 67) and for the median age (46) of the study are provided in Figure 2.

Figure 2 does not show an increase in average performance for the more recent cohorts; therefore, calculation (addition) skills were not affected by the Flynn effect. This observation is even more interesting in the light of the work presented by Salthouse (2005, p. 552). Some of Schaie's (2005) results, which were reported by Salthouse (2005) using a similar format to our Figure 2, showed that


Figure 1. Mean arithmetic scores for Norwegian conscripts, all with the same age, plotted against the year of the test (adapted from Table 1 and Figure 2 of Sundet et al., 2004).


Figure 2. Mean performances for the verification of simple additions for three age groups plotted against the year of the test (based on data from Schaie, 2005).
the Flynn effect influences other processes, such as inductive reasoning (completing series), for which there were almost systematic increases in the scores of the more recent cohorts.

Thus, the research carried out by Sundet et al. (2004) and by Schaie (2005) suggested that arithmetic cognition is not subject to the Flynn effect (superior performance of recent cohorts), at least for the period covered by these two studies (second half of the 20th century). Consequently, we concluded that if a cross-sectional study covering that period showed a reduction in arithmetic performance with age, it probably would not be due to a Flynn effect.

## Pragmatic Cognition Versus Mechanical Cognition

Our second concern was to determine the effect of age on solving arithmetic problems: an effect that depends on the type of cognition involved. Researchers (e.g., Cattel, 1971) have long made a distinction between fluid cognition (i.e., mechanical) and crystallized cognition (i.e., pragmatic) or, more recently, between mechanical cognition and pragmatic cognition (Baltes et al., 1999). In adults, mechanical (fluid) cognition is more subject to the negative effects of aging than pragmatic (crystallized) cognition, which, in certain domains, such as cultural knowledge, can continue to progress throughout a person's lifespan (Baltes et al., 1999). However, to the best of our knowledge, the cognition underlying numeracy has never been explicitly identified. Before attempting to determine the type of cognition used in solving arithmetic problems, the distinction between these two cognitions must be accurately defined.

Baltes et al. (1999) showed that pragmatic cognition is associated with culture and formal learning. Lövdén, Ghisletta, and Lindenberger (2004) more precisely defined pragmatic cognition as the acquisition and expression of bodies of declarative and procedural knowledge that are transmitted culturally and that are available to individuals during socialization. Age-related reductions in performance tend to be attenuated for knowledge-rich domains with everyday relevance, such as practical problem solving, social cognition, memory in collaborative contexts, life planning, wisdom (no negative difference up to the age of 75), interactive-minds cognition, and card playing (Baltes et al., 1999). Furthermore, Schaie (2005) explicitly considers numerical skills as being representative of crystallized intelligence.

In their description of mechanical (fluid) cognition, Baltes et al. (1999) maintain that cognitive mechanics are indexed by the speed, accuracy, and coordination of elementary processing operations. This type of cognition is profoundly influenced by
the biological conditions affecting a person. The predominant lifespan pattern of this type of cognition can be divided into successive periods: maturation (from early childhood to adulthood), stability (around 25 years of age), and aging-induced decline (which starts at around 30). More specifically, Lövdén et al. (2004) believe that the ability to reason in highly over-learned or novel domains is a property of mechanical cognition. Schaie (2005), on the other hand, favors the idea that inductive reasoning (e.g., completing a series of letters, such as "abxcdxefxghx...") represents fluid intelligence. Cattell (1971), for whom sequential reasoning is crystallized intelligence, went as far as classifying quantitative reasoning in this same category. Hence, we saw that reasoning, at least in some of its forms, can clearly be categorized as mechanical cognition (fluid).

In conclusion, even if we accept the classic distinction between the two types of cognitionmechanical (fluid) versus pragmatic (crystallized)our current theoretical knowledge does not allow us to establish unambiguously which of the two is most important in solving arithmetic problems. Consequently, it is difficult to predict how mental arithmetic abilities are likely to evolve with age and, therefore, there is a need to assess this evolution empirically.

## Method

## Survey Procedure

The IVQ 2004 study was based on a sample of 17,500 homes selected at random by the INSEE. A quarter of the homes were empty when the survey was conducted. Of the homes that were occupied ( $75 \%$ of the sample), $20 \%$ of the occupants refused to participate in the survey, resulting in a final sample of 10,384 people (only one person was surveyed in each home). Each participant took 50 to 60 minutes to complete the survey.

The arithmetic tests were administered in two parts (with a total duration of less than 10 minutes). The first part was carried out after a number of exercises to evaluate the participant's overall literacy level and involved the participant reading two numbers and solving simple arithmetic problems (referred to as "orientation problems"). The second group of arithmetic tests was presented at the end of the survey, after the reading skills exercises. The participant was set a sequence of 11 arithmetic problems, presented in order of increasing difficulty. If the participant had shown a low level of performance during the last three of the five orientation problems, the sequence was started at the beginning. Otherwise, the interviewer moved straight
on to the second part of the sequence (see Appendix A). After three mistakes (consecutive or not), the test was stopped. The participant could ask the interviewer to repeat a problem up to six times, and the number of repetitions was noted. If, after approximately two minutes, the participant did not provide an answer, the interviewer moved to the next item.

The problems (5 orientation plus sequence of 11) alternately examined additive structures (comparison of sizes, calculation of differences), multiplicative structures (multiplication, division with or without remainders, finding the fourth proportional number), calculating percentages (prize giving), and/or were based on class logic (with the standard operations of union, intersection, and complement). Illustrative examples of the problems are presented in Appendix A.

## Participants

Our analysis was based on the 9,185 participants for whom complete sets of data were obtained. The individuals were divided into 18 groups, created by combining three levels of education (at least elementary school level, secondary school level, and higher education level) with six age groups (18-25, $26-33,34-41,42-49,50-57$, and 58-65). The age statistics for the resulting 24 groups are given in Appendix B. The median and statistically significant relationship between age and level of education (Cramer's $V=0.39, X^{2}(10)=2700.98, p<.001$ ) was mostly due to the low and high levels of education (the first was over-represented in the older age groups and the second was over-represented in the younger age groups). The secondary school level was almost independent of age.

## Performance Measures

Performances were measured according to the number of correct answers (one point per item, giving a maximum score of 16 ), and the average number of repetitions for each problem. Scores were calculated assuming that participants who started the test at the second part of the sequence would have successfully solved all the problems in the first part. It was also assumed that the questions not presented (because the participant already had given three incorrect answers) would have been answered incorrectly. This calculation method was strictly identical to the method used in, and validated by, a preparatory study for IVQ 2004 (Charron et al., in press). The average number of repetitions was calculated by dividing the total number of repetitions each participant requested by the number of problems presented to that participant. Therefore, for a same score, the lower the average number of repetitions, the better the performance.

## Hypotheses

The above theoretical considerations were taken into account when formulating our hypotheses. We have already observed (Charron et al., in press) that the arithmetic items drawn up for the IVQ survey address skills that have always been taught in school and relate to omnipresent themes in our culture (e.g., buying, temperature, transportation). In addition, the form in which the items were presented and the way in which the answers were given were reminiscent of the presentation of mental arithmetic problems at school (cf. Appendix A). Because all the IVQ items considered in this study involved numerical processing, we hypothesized that solving these arithmetic problems relies, at least partially, on pragmatic cognition. However, solving these problems also may involve mechanical cognition, as several of them require reasoning, which, as we have already seen, is partially underlain by mechanical cognition, especially when assessing new domains.

Consequently, the evolution in arithmetic performances for the age span in question (18-65) should depend on the relative importance of each type of cognition in processing the items. If the cognition involved is mostly of the pragmatic type, performances should remain stable or improve slightly. Alternatively, if mechanical cognition is predominant, performances would be expected to deteriorate with age.

The effect of age also may depend on level of education. In fact, an increasing number of researchers consider level of education to be crucial in cognitive tasks that are subject to deterioration with age (Corral, Rodriguez, Amenedo, Sanchez, \& Diaz, 2006; Ska, Schroeders, Poissant, \& Joanette, 2000). Deterioration in performing these tasks is less marked in persons with a high level of education. A longitudinal study on a sample of 1,189 participants, conducted by the McArthur Foundation in the United States (reported by Fontaine and Toffart, 2000), also showed that level of education was the best predictor of optimal aging from a cognitive point of view (i.e., maintenance of a high functional level). Although the IVQ survey population was a little young to be considered "aging," we expected the evolution of performance with age (in the sense of whether performances remain steady or decline) to be better for higher levels of education and poorer for lower levels of education. In particular, we expected that individuals with a low level of education are more likely to have recourse to reasoning, otherwise known as fluid cognition (which is sensitive to the negative effects of age from 30 onwards). This is because they have not gained as much as have the others from the cultural benefits of schooling and, therefore, they cannot rely on previously learned
ready-made answers in order to solve the problems presented.

Data from a preparatory study for the IVQ (Charron et al., in press) have already provided some answers to these questions. However, these data are quantitatively insufficient for testing finer hypotheses, such as the effect level of education has on the evolution of numeracy with age. This preparatory survey showed a decline in arithmetic performance between the ages of 18 and 65 . Most importantly, it also showed that level of education is the greatest factor in explaining arithmetic performance: performance increases with level of education. In France (as in many other countries), the number of years spent at school or in higher education increased significantly during the second half of the 20th century (Direction de l'Evaluation, de la Propective et de la Performance [DEPP], 2007). Consequently, young people are more likely to have a high level of education and older people are more likely to have a very low level of education. This link between age and level of education must be taken into account when analyzing the evolution of performances. This was the approach we adopted when carrying out our research. However, we also attempted to determine the effect of age independent of level of education.

## Comments

Because the age range covered by the IVQ
survey (18 to 65 years old) was quite small for a lifespan study, it is important to highlight a number of points. First, it is extremely difficult to obtain a sample that is representative of all socio-educative levels for people at an advanced age (e.g., 80), because it is a demographic fact that individuals from higher social classes tend to live longer than do others (Institut National de la Statistique et des Études Économiques [INSEE], 2005). In addition, among the survivors at an advanced age, it is difficult to test those with weak cognitive functioning (e.g., due to illness, lack of motivation). For example, Yang, Krampe, and Baltes (2006) have pointed out that the 68 members of their sample who were over the age of 70 represented, from a cognitive functioning point of view, the upper two thirds of their age cohort.

Second, similarly, at a young age (e.g., 18), it is difficult to find people with a low level of education (elementary level or less), because school is mandatory until adolescence. Third, imaging, has shown a gradual neurological decline between the ages of 20 and 80 (Grady, Springer, Hongwanishkul, McIntosh, \& Winocur, 2006). Because the very large survey sample provided by IVQ 2004 gave our study great statistical power, we hoped we would be able to detect slight changes that have so far gone undetected in behavioral studies and that would reflect the results of Grady et al.’s neurological study.


Figure 3. Evolution of performance with age.

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Figure 4. Evolution of performance with level of education.

## Results

## Analysis of Performance According to Age and Level of Education

Our first step was to examine the descriptive results, which are presented in Appendix C. The graphs of the average scores and the average number of repetitions for each age group (Figure 3) show a systematic decrease in scores with increasing age and a small increase in the number of repetitions. However, this preliminary analysis does not take into account the differences in level of education, which have a considerable influence on scores and on the average number of repetitions (Figure 4). Average scores are significantly higher and average numbers of repetitions are lower for participants with higher levels of education, thereby indicating a marked increase in performance with level of education. This result confirms the findings of the preparatory study (i.e., Charron et al., in press).

Figures 5 (scores) and 6 (number of repetitions) present a more detailed breakdown of the evolution of performance with age for each level of education. Figure 5, which shows the relationship between average scores and age for each level of education, indicates that performances did not decrease with age for any of the levels of education. For participants in the secondary and higher education groups, scores
remained stable across all the age groups; whereas for the lowest level of education, there was a marked increase in scores with age, although they never reached the levels recorded for the higher levels of education. ${ }^{3}$ The average numbers of repetitions, shown in Figure 6, were homogenous across the age groups for all three levels of education, with the exception of the first two age groups of the elementary level. These two age groups required the lowest and highest numbers of repetitions, respectively.

Stage 2 of our analysis involved measuring the observed effects and generalizing them for the parent population. Because our initial analyses indicated a linear relationship between the averages recorded, it was possible to model the effects obtained using the slopes of regression lines. In order to neutralize any possible covariance between the score and the number of repetitions, we carried out a multivariate Bayesian analysis of the comparisons for the S<A6 $\times$ L3> design, where S represents the Subject factor, A represents the Age factor (with the modalities a1 to a6 representing the age groups in ascending order), and $L$ represents the Level of Education ( $11=$ elementary, $12=$ secondary, $13=$ higher). This method estimates the size of the parent effects (Lecoutre, 1984; Lecoutre \& Poitevineau,


## Age group

Figure 5. Evolution of scores with age and level of education.


Age group

Figure 6. Evolution of mean number of repetitions with age and level of education.

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2007; Rouanet, 1996, 1998) from the distribution of the sample. Table 1 shows the slope of the observed effects, as well as the upper and lower limits estimated for the true effects (in the "Bayesian statement" column). Analysis of Table 1 indicates that performances in the parent population were generally poorer among older participants and improved with level of education. The true effects are significant and positive according to the Bayesian statements, which estimate them to be greater than 0.07 for age and greater than 0.54 for level of education (Lecoutre, 1984). The Bayesian statements also show an interaction between age and level of education, although this is solely due to the improvements in performance with age recorded for participants with an elementary level of education. According to the Bayesian statement, the true effect of age for the elementary level of education is significant and greater than 0.14 , whereas the performances of the two higher levels of education remain stable with age, with non-significant true effects that the Bayesian statements situate between -0.02 and 0.02 in both cases.

These statistical results confirm the expected positive effect of level of education, but they also show that the age effect was more favorable for
participants with the lowest levels of education, as they were the only participants to show improved performance between the ages of 18 and 65 . This second result was unexpected. However, the results do not allow us to determine the general effect of age on performance independent of level of education.

## Analysis of the Effect of Age Independent of Level of Education

The next stage was to attempt to reveal the main effect of age by neutralizing the relation between age and level of education. In order to do this, we carried out a multivariate Bayesian analysis of the comparisons of the slopes for the $\mathrm{S}<\mathrm{A} 6>$ design (where S represents the Subject factor and A the Age factor), taking into account the level of education by using co-variables (Lecoutre, 1984; Lecoutre \& Poitevineau, 2007; Rouanet, 1996, 1998). These covariables correspond to the elementary-level and higher-level indices. ${ }^{4}$ As above, this analysis allowed us to determine the sign and magnitude of the slopes for the true effects of age on the score and average number of repetitions variables for the parent population after the influence of level of education had been eliminated.

Table 1
Multivariate Analysis of the Comparisons of the Standardized Slopes ${ }^{a}$ of the Means

|  | Description |  |  |  | Inference (on the global effect) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Slope of scores | Slope of repetitions | Global effect | Residual | $F^{b}$ | $d f^{e}$ | Bayesian statement ${ }^{\text {d }}$ |
| A | -0.11 | 0.03 | 0.08 | 0.03 | 149.99* | 9166 | > 0.07 |
| L | 0.77 | -0.16 | 0.56 | 0.02 | 1174.46* | 9166 | $>0.54$ |
| A/l1 | 0.23 | -0.01 | 0.17 | 0.06 | 37.77* | 1328 | $>0.14$ |
| A/l2 | 0.01 | 0.01 | 0.01 | 0.06 | 1.42 | 5201 | [-0.02, 0.02] |
| A/l3 | 0.01 | 0.01 | 0.01 | 0.04 | 0.42 | 2635 | [-0.02, 0.02] |

${ }^{\mathrm{a}}$ The analysis is based on the standardized slopes, that is to say, the ratio of the slopes to their standard deviations. As these standardized slopes are ratios, they are all comparable to each other. For the calculations, the abscissas for groups a1, a2, a3, a4, a5, and a6 are, respectively, 1, 2, 3, 4, 5, and 6.
${ }^{\mathrm{b}}$ The $F$ were calculated assuming equal intragroup variances in the population.
${ }^{\text {C }}$ The first degree of freedom was systematically 2.
${ }^{\mathrm{d}}$ The Bayesian statements were calculated for the usual limit of .90.

* Statistically significant values at alpha $=.001$


## Table 2

Univariate Analyses of the Variable Slopes Adjusted for the Covariates ${ }^{a}$

|  | Description |  |  |  | Inference (on the slope) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | slope | standardized slope | residual | standardized residual | $t^{b}$ | $d f$ | Bayesian statement ${ }^{\text {C }}$ |
| score | 0.10 | 0.04 | 0.21 | 0.08 | 5.07* | 9177 | > 0.07 |
| repetition | 0.005 | 0.01 | 0.01 | 0.02 | 1.50 | 9177 | [-0.01, 0.01] |

${ }^{a}$ For the calculations, the abscissas of groups a1, a2, a3, a4, a5, and a6 were, respectively, 1, 2, 3, 4, 5, and 6 .
${ }^{\mathrm{b}}$ The $t$ were calculated assuming equal intragroup variances in the population.
${ }^{\text {c }}$ The Bayesian statements were calculated for the usual limit of .90 .

* Statistically significant values at alpha = . 001

Table 2 summarizes the results for each variable. Age produced a noticeable improvement in scores: According to the Bayesian statement, its slope was statistically significant and greater than +0.07 . On the other hand, the true effect of age on the average number of repetitions was small. It was statistically non-significant and had a true value, according to the Bayesian statement, of between -0.01 and +0.01 . Therefore, it can be considered negligible. A complementary multivariate analysis confirmed the results shown in Table 2 (the respective values of the standardized effects were identical), as well as the existence of a positive (+0.03) and statistically significant global standardized effect of age, $F(2$, 9176 ) $=13.22, p<.0001$.

The effect of age is illustrated in Figure 7, which shows, for each age group, the average scores adjusted for the co-variables (details of the statistics are given in Appendix D). These adjusted scores suggest that the effect of age is globally positive: after a slight decrease in performance in the median age groups, the adjusted means given in Figure 7 increase markedly for the upper two age groups. In order to show clearly that a failure to take into account the effect of level of education leads to an opposite conclusion about the effect of age, Figure 7 includes the non-adjusted mean scores (already shown in Figure 3). Thus, rather than a decrease in performance with age, as initially suggested by the non-adjusted means, there is, in fact, a nonmonotonic pattern with a final increase in performance (as indicated by the means adjusted for level of education).

To eliminate the possibility that the results of our analyses of the effects of age were an artifact of the age groups into which the participants were divided,
all the statistical analyses were repeated with the participants divided into 48 age groups (one group per year). The results were confirmed without the slightest deviation.

## Discussion

This research has produced three important results. First, for French adults aged 18 to 65, performances in solving arithmetic problems generally appeared to be poorer in the oldest age groups. However, in reality, performances were seen to improve with age when the level of education factor was eliminated. Second, performances in solving arithmetic problems improved with increasing level of education. Third, for individuals with higher and secondary levels of education, performances were stable across all the age groups, whereas the performances of participants with an elementary level of education improved with age, although their performances never reached the standard attained by individuals with higher levels of education.

At first sight, the age-related decrease in arithmetic problem-solving performance, a skill that does not appear to be subject to the Flynn effect (see earlier), suggests a cognitive decline with age. However, when the level of education variable is neutralized, the results of our analyses suggest that such an interpretation is too simplistic. In fact, when persons with similar levels of education are compared, or when the covariance between age and level of education is removed from the effect of age, the evolution is seen to be more towards stability or improvement with age. Therefore, the decline in performance with age that was seen in the crosssectional analysis of the mean scores (that is to say,

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Figure 7. Evolution of scores with age for the adjusted (for level of education) means and non-adjusted means.
without taking into account the level of education) was only a demographic characteristic of the population of France, in that younger people have generally spent more time in education than have their elders. So, we can add the cognition underlying arithmetic processes to Baltes et al.'s (1999, p. 492; also see our theoretical analysis in the initial presentation of the problem) list of fundamentally pragmatic skills that barely decline with age.

Obviously, this first conclusion cannot be definitive. For example, we cannot be certain that the teaching provided by the French education system from the 1940s to the end of the 20th century has provided calculation and numerical skills that should lead to identical performances in solving arithmetic problems. If the skills acquired immediately after the war were more robust than were those required during later periods, any negative effect of age would be masked. Conversely, an improvement in the effectiveness of the teaching also could mask any positive effect of age, as the performances of the younger people would be artificially "boosted" by this better teaching. It would be interesting to reexamine the graph of performance against age (the graph of adjusted means, see Figure 7) in conjunction with a thorough analysis of the curricula and teaching methods used over the years. Such an analysis has been carried out for the teaching methods used in the United States (Blair, Gamson, Thorme, \& Baker,
2005), but its conclusions do not throw much light on our data. In fact, according to Blair et al. (2005), the greatest effect of the expansion of schooling has been to raise the level of fluid intelligence. In our study, this neither explains the evolution of arithmetic cognition (accepting that this skill relies on pragmatic knowledge), nor does it explain the progress of the participants with a low level of education, who only attended school for a very short period.

The second result confirms the hypothesis that performances should improve with level of education. At all ages, participants who went on to higher education perform better than do those who left school after the secondary level, who, in turn, perform better than those with only an elementary level of education. This effect is seen in the scores of the participants and in the mean number of repetitions of the problems. If effectiveness is defined as the ability to arrive at the correct answer and if ease is defined as the ability to achieve this answer with few repetitions, then individuals with a high level of education are more effective and more at ease. The link between performance and level of education is probably just as much a result of the skills learnt as it is a consequence of intellectual efficiency (Charron et al., in press). Firstly, the schooling the participants received provided them with the skills needed to process problems. These include mathematical skills and the skills needed to
manage best the evaluation situation in which they were placed during the IVQ 2004 survey (some participants probably felt as if they were in an examination situation). Secondly, level of education may have been partially confounded with true intellectual ability, either because the education system eliminates, ipso facto, the weakest students, or because studying contributes to the development of the short-term memory, which is particularly useful for processing problems (as Cole, 2005, has shown from a meta-analysis of experimental studies), especially when they are stated orally, as was the case in the IVQ survey.

The third result was completely unexpected. Because many researchers (e.g., Corral et al., 2006) have suggested that level of education may be a protection factor against the expression of cognitive decline, we expected the evolution of performance with age for the participants with the lowest level of education to be less favorable than for the others. This is not what was observed. We did observe the expected maintenance of performance levels for participants with secondary or higher levels of education, but the arithmetic performances of the participants who only had an elementary level of education (or even less) improved regularly with age. However, this, the most important and most original result of our study, needs to be confirmed, as there was only a modest number of participants in certain age groups and because the elimination of participants for whom we did not have full sets of data did not affect all age and education groups randomly. To the best of our knowledge, such a progression with age (at least up to 65 years old) for individuals with low levels of education has never before been observed; lifespan studies generally show, at best, the maintenance of a level of performance with increasing age. Of course, such a surprising result needs to be interpreted, but it is also necessary to determine why improvements in performance with age for participants with low levels of education have not been detected before.

If the arithmetic skills taught at elementary school are considered to have remained more or less equivalent over the years (which is not at all certain), we can suppose that the improvements in performance of participants with an elementary level of education (or less) are due to day-to-day activities. Such a development of mathematical skills with age, outside the education system, has been recorded for populations that have had little or no formal education, for example, for Brazilian candy sellers in Récife and sugar cane planters in Nordeste (Fischer, 2002; Saxe, 1998). This development is due to socially organized, day-to-day activities in which mathematical skills are needed to solve problems
related to important domains, such as achieving economic goals. However, this involves increasing practical skills rather than developing true mathematical expertise (Fischer, 2002). It seems likely that a similar developmental mechanism is at work in adults with low levels of education: the development of skills through social and professional activities partially compensates for the lack of school learning, but this development is not sufficient for them to attain the same level of performance as adults with a higher level of education. It would be interesting to carry out a detailed qualitative study of the skills of people with a low level of education, in order to determine how these skills differ from those of people with a high level of education. Whatever its cause, the improved performances of people with a low level of education strengthen our belief that the processing of arithmetic problems (at least those with a scholarly format, as in the IVQ survey) relies more on pragmatic cognition than on mechanical cognition.

If, as seems to be the case, day-to-day activities play a role in the development of arithmetic skills, why do we not see an improvement in the skills of people with secondary and higher levels of education? We can immediately eliminate the idea that these two groups have attained a ceiling score, as their means are far below the possible maxima. The hypothesis that these individuals have attained the pinnacle of their development and are therefore unable to develop their mathematical skills any further, also seems unlikely, as a series of studies carried out by Baltes and his colleagues at the Max Plank Institute (e.g., Yang et al., 2006) show that adults, including those over 65, have reserves of cognitive abilities. These reserves mean that skills can improve continuously, given suitable cognitive training. Consequently, it is possible that day-to-day cognitive activities are not sufficient to activate this cognitive reserve, as the ability level of people with a high level of education is sufficient for them to be able to cope with most of the arithmetic situations they regularly have to face. If this is the case, the reason people with secondary or higher levels of education do not progress is not because they are unable to do so, but rather because they do not need to do so for their day-to-day activities.

As far as we are aware, the progression with age of people with low levels of education has not been observed before. One possible reason for this nonobservation is undoubtedly methodological, as the over-representation within a sample of individuals with high levels of cognitive functioning is quite systematic and difficult to avoid with older cohorts. The research carried out by Yang et al. (2006) and briefly described in the "Survey" section of this paper, provides a good example of this type of bias.

Another example, also mentioned above, is the recent study by Grady et al. (2006), in which some of the participants initially recruited were eliminated because their scores on the mental status examination were too low. According to the authors, these low scores were related to their low levels of education. Once the low-scoring participants had been eliminated, the remaining participants were divided into three groups with mean ages of 23.2, 46.5, and 74.4. The average numbers of years of education for the members of each group were almost identical, at $14.8,14.9$, and 14.9 years, respectively. The 16 adults ( 8 men and 8 women) in the group with a mean age of 74.4 would have completed their university studies during the 1940s and 1950s. However, the graph produced by Blair et al. (2005, p. 98) shows that such a sample cannot be considered representative of its cohort from the point of view of years of education. In fact, it falls into the upper 10th of the cohort. Thus, the specific developmental phenomena of participants with low, or even average, levels of education are likely to be missed by this type of research. Our results strongly suggest that such phenomena exist.

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## Appendix A - Typical examples of items

Remark: As we do not have the right to publish the original items, ${ }^{1}$ the examples presented below are representative of and very similar to the items used in the IVQ 2004 survey.

1) Typical examples for the "Orientation" section:
"Please, read what is written. Off you go" [The pollster points to the number 5489, which is written in Arial 48 on a $10.5 \times 14.5 \mathrm{~cm}$ card. The response is noted as given].

In Bolivia, the temperature is $-10^{\circ}$ at night and $+35^{\circ}$ during the day. What is the difference in temperature between night and day?
2) Typical examples for the items at the beginning of the chain:

At the bus stop, 8 people get on the bus and nobody gets off. The driver counts the passengers. There are now 24 . How many passengers were there on the bus before it got to the stop?

A customer is trying to work out the price of a t-shirt from a torn label. The person knows a pack of 8 costs $€ 56$. How much does one t-shirt cost?
3) Typical examples for the second part of the chain:

At a train station, 8 people get on and 13 get off. When the train sets off, are there more or less passengers on board? How many?

In a sale with a $40 \%$ discount, a sofa finally costs $€ 300$. How much was reduction from pre-sale price?

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Appendix B - Descriptive statistics for the ages of each group

| Group | $n$ | $M$ | $S D$ | Min | Max |
| :---: | :---: | :---: | :---: | :---: | :---: |
| a111 | 13 | 22.00 | 1.92 | 19 | 25 |
| a112 | 643 | 21.45 | 2.37 | $17^{*}$ | 25 |
| a113 | 574 | 21.85 | 2.12 | $17^{*}$ | 25 |
| a211 | 27 | 29.44 | 2.28 | 26 | 33 |
| a212 | 853 | 29.91 | 2.35 | 26 | 33 |
| a213 | 765 | 29.74 | 2.27 | 26 | 33 |
| a311 | 52 | 37.39 | 2.12 | 34 | 41 |
| a312 | 1215 | 37.54 | 2.32 | 34 | 41 |
| a313 | 519 | 37.16 | 2.23 | 34 | 41 |
| a411 | 150 | 46.53 | 2.24 | 42 | 49 |
| a412 | 1044 | 45.33 | 2.33 | 42 | 49 |
| a413 | 355 | 45.29 | 2.29 | 42 | 49 |
| a511 | 472 | 53.84 | 2.22 | 50 | 57 |
| a512 | 841 | 53.45 | 2.33 | 50 | 57 |
| a513 | 261 | 53.28 | 2.30 | 50 | 57 |
| a611 | 621 | 61.67 | 2.34 | 58 | 65 |
| a612 | 612 | 61.30 | 2.33 | 58 | 65 |
| a613 | 168 | 60.42 | 2.10 | 58 | 65 |

$\overline{\text { Notes. } N=9,185 \text {. The codes a1 to a6 correspond to the age groups: respectively from 18-25 years old to 58-65 }}$ years old. The codes $11,12,13$ correspond to the level of education: respectively, elementary (or lower), secondary, and higher.

* A very small number of participants were only 17 years old; however, they were only a few days away from their 18th birthday and were therefore included in the $18-25$ age group.

Appendix C - Descriptive statistics of performances

1) By age group

|  | $18-25$ | $26-33$ | $34-41$ | $42-49$ | $50-57$ | $58-65$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| score |  |  |  |  |  |  |
| $M$ | 11.74 | 11.61 | 11.13 | 10.73 | 10.67 | 10.48 |
| $S D$ | 2.41 | 2.66 | 2.85 | 3.09 | 3.14 | 3.05 |
| $M$ | 1.70 | 1.73 | 1.71 | 1.73 | 1.73 | 1.76 |
| $S D$ | 0.48 | 0.49 | 0.52 | 0.51 | 0.50 | 0.50 |
| $\min$ | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| $\max$ | 4.00 | 4.00 | 6.00 | 4.60 | 4.20 | 4.00 |

Note. For the score, the $\min$ and the $\max$ values were always 0 and 16. The mean standard deviations for the scores and the repetitions were 2.60 and 0.50 , respectively.
2) By level of education

|  | elementary | secondary | Higher |
| :--- | :---: | :---: | :---: |
| $M$ | 8.66 | Score |  |
| $S D$ | 3.55 | 10.83 | 12.71 |
|  |  | 2.68 | 1.74 |
| $M$ | 1.76 | Repetitions |  |
| $S D$ | 0.53 | 1.74 | 1.69 |
| $\min$ | 1.00 | 0.51 | 0.47 |
| max | 4.60 | 1.00 | 1.00 |

Note. For the score, the $\min$ and the max values were always 0 and 16, except for the higher level, where the min was 3 . The mean intragroup standard deviations for the scores and the repetitions were 2.60 and 0.50 , respectively.
3) Means and standard deviations (in parentheses) of the scores by age and level of education

|  | $18-25$ | $26-33$ | $34-41$ |  | $42-49$ | $50-57$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| elementary | $4.77(5.04)$ | $6.82(4.57)$ | $6.25(4.07)$ | $7.43(3.80)$ | $8.77(3.57)$ | $9.24(3.34)$ |
| secondary | $11.09(2.41)$ | $10.72(2.74)$ | $10.70(2.77)$ | $10.51(2.86)$ | $11.09(2.63)$ | $11.16(2.41)$ |
| higher | $12.64(1.74)$ | $12.78(1.75)$ | $12.64(1.82)$ | $12.78(1.62)$ | $12.76(1.64)$ | $12.63(1.89)$ |

4) Means and standard deviations (in brackets) of the numbers of repetitions by age and level of education

|  | $18-25$ | $26-33$ | $34-41$ | $42-49$ | $50-57$ | $58-65$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| elementary | $1.52(0.50)$ | $1.87(0.70)$ | $1.73(0.64)$ | $1.73(0.55)$ | $1.74(0.51)$ | $1.78(0.53)$ |
| secondary | $1.74(0.51)$ | $1.75(0.51)$ | $1.73(0.53)$ | $1.74(0.52)$ | $1.75(0.50)$ | $1.76(0.47)$ |
| higher | $1.66(0.44)$ | $1.70(0.46)$ | $1.68(0.49)$ | $1.71(0.48)$ | $1.67(0.49)$ | $1.72(0.51)$ |

Appendix D - Univariate descriptive statistics for the variables adjusted for level of education

|  | $18-25$ | $26-33$ | $34-41$ | $42-49$ | $50-57$ | $58-65$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| scores (adjusted by the primary and higher levels) |  |  |  |  |  |  |
| $M$ | 11.09 | 10.72 | 10.69 | 10.51 | 11.09 | 11.16 |
| $S D$ | 2.17 | 2.38 | 2.58 | 2.74 | 2.82 | 2.81 |
| $M$ |  | repetitions | (adjusted by the primary and higher levels) |  |  |  |
| $M D$ | 1.74 | 1.75 | 1.73 | 1.74 | 1.75 | 1.76 |
|  | 0.48 | 0.49 | 0.52 | 0.51 | 0.50 | 0.50 |

Note. The mean intragroup standard deviations were 2.89 for the scores and 0.50 for the repetitions. The mean intragroup correlation was 0.10 . The mean intragroup covariance was 0.17 .

1 The Information and Vie Quotidienne (IVQ) survey was organized by the INSEE in collaboration with the National Agency for the Fight Against Illiteracy (ANLCI); the Center for Economic and Statistical Research (Crest); the Directorate for Research, Studies, and Statistics (Dares) and the General Delegation for Employment and Professional Training (Dgefp)—both part of the Ministry of Employment and Social Cohesion; the Directorate for Assessment and Forward Planning (DEPP) of the Ministry of Education and Research; the Directorate for the French Language and the Languages of France (Dglflf) of the Ministry of Culture; the Interministerial Agency for Urban Affairs and Social Development (Div); the National Institute for Demographic Studies (Ined); and the National Poverty and Social Exclusion Observatory.
2 At first glance, it may be tempting to consider the scores achieved on the test to be measures of numerical abilities and the average number of repetitions of the problems to be measures of short-term memory. In fact, in this case, the two parts cannot be dissociated, as the number of times a problem is repeated can have an incidence on whether or not it is answered correctly. In addition, the repetitions score may be a measure of short-term memory span linked to oral comprehension, to calculations, or to reasoning, or it may be an indication of the degree of familiarity with the tasks being presented. Finally, this score may simply indicate some sort of deficiency (hearing, attention), whether this deficiency is permanent or temporary (for example, due to a problem during the test).
${ }^{3}$ At first sight, the reader might be surprised by the fact that the global effect of age is negative (Figure 3), whereas the effect of age for each level of education is positive or stable (Figure 5). This result comes from the interaction between the unbalanced members and the variable scores of the groups (see Appendices B and C).
4 As the level of education variable was a non-numerical ordinal variable with three levels (elementary, secondary, and higher), it was coded into two indices. As a reminder, all the information for a variable with $n$ modalities is contained in $n-1$ binary indices.


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